The Polytropic Process – Ideal Gas

Definition of a polytropic process: $$\frac{p_2}{p_1} = \left(\frac{\forall_1}{\forall_2} \right)^n$$

If the fluid is an ideal gas,

$$\frac{p_2}{p_1} = \left(\frac{\forall_1}{\forall_2} \right)^n = \left(\frac{mRT_1 / p_1}{mRT_2 / p_2} \right)$$

This leads to two additional relationships for **ideal gases**,

$$\frac{T_2}{T_1} = \left(\frac{p_2}{p_1} \right)^{(n-1)/n} \quad \text{and} \quad \frac{T_2}{T_1} = \left(\frac{\forall_2}{\forall_1} \right)^{1-n}$$

The Polytropic Process – Ideal Gas

The work done during a polytropic process is,

$$W_{12} = \int_{\mathcal{V}_1}^{\mathcal{V}_2} \frac{p_1 \mathcal{V}_1^{\,n}}{\mathcal{V}^{\,n}} d\mathcal{V} = \frac{p_2 \mathcal{V}_2 - p_1 \mathcal{V}_1}{1-n} \quad \text{for} \quad n \neq 1$$

If the fluid is an ideal gas,

$$W_{12} = \frac{mR\left(T_2 - T_1\right)}{1-n} \quad \text{for} \quad n \neq 1$$

For the case where n = 1,

$$W_{12} = \int_{\mathcal{V}_1}^{\mathcal{V}_2} \frac{p_1 \mathcal{V}_1}{\mathcal{V}} d\mathcal{V} = p_1 \mathcal{V}_1 \ln \frac{\mathcal{V}_2}{\mathcal{V}_1}$$

Ideal Gas Entropy → f(T,V)

For the ideal gas, recall that

$$Pv = RT, \quad du = c_v dT, \quad \text{and} \quad dh = c_p dT$$

Then from the first Gibbs equation,

$$s_2 - s_1 = \int_{u_1}^{u_2} \frac{du}{T} + \int_{v_1}^{v_2} \frac{P}{T} dv = \int_{T_1}^{T_2} c_v \frac{dT}{T} + \int_{v_1}^{v_2} \frac{RT}{v} \frac{dv}{T} = \int_{T_1}^{T_2} c_v \frac{dT}{T} + R \ln \frac{v_2}{v_1}$$

Furthermore, if the heat capacity can be assumed constant,

$$s_2 - s_1 = c_v \ln \frac{T_2}{T_1} + R \ln \frac{v_2}{v_1}$$

Notice that the entropy of an ideal gas is a function of *both* temperature and specific volume (or pressure).

Ideal Gas Entropy →f(T,P)

For the ideal gas, recall that

$$Pv = RT, \quad du = c_v dT, \quad \text{and} \quad dh = c_p dT$$

Then from the second Gibbs equation,

$$s_2 - s_1 = \int_{h_1}^{h_2} \frac{dh}{T} - \int_{P_1}^{P_2} \frac{v}{T} dP = \int_{T_1}^{T_2} c_p \frac{dT}{T} - \int_{P_1}^{P_2} \frac{RT}{P} \frac{dP}{T} = \int_{T_1}^{T_2} c_p \frac{dT}{T} - R \ln \frac{P_2}{P_1}$$

Furthermore, if the heat capacity can be assumed constant,

$$s_2 - s_1 = c_p \ln \frac{T_2}{T_1} - R \ln \frac{P_2}{P_1}$$

Notice that the entropy of an ideal gas is a function of *both* temperature and pressure.